Bathtub Redesign

"immerse yourself ..."

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Table of Contents

1. Introduction

Whether used for cleaning purposes or just relaxing, bathtubs exist in most households around the world. "In the past half century, the number of bathrooms per American has doubled" [1]. Every home has at least one bathroom, which makes showers/tubs a commonly purchased product.

A study differentiated full immersion bath, shower, mist sauna, and no bathing with correlations to recovery from local muscle fatigue. The results showed that the mean power frequency (MNF) of the electromyogram (EMG) for the bathtub (full immersion bath) was notably higher than the no bathing situation. This data translates to a good recovery from muscle fatigue [2]. Furthermore, another paper concurs by stating: "Routine immersion bathing appeared more beneficial to mental and physical health than routine shower bathing without immersion" [3]. Immersive showering (bathtubs) is the main topic for this paper. The benefits expand to bathing small children, great for relaxing sore muscles, available features such as air jets or whirlpool ... [4].

Accommodation is essential, especially with the size differences. For example, big and tall individuals will face discomfort when staying at other houses or hotels/Airbnb. Specifically, they will have to bend their knees rather than lay flat. Additionally, width might also come as an issue. As a solution, a one-size-fits-all bathtub design has to accommodate most of the population.

In the next part of this paper, the study will be explained in detail and performed.

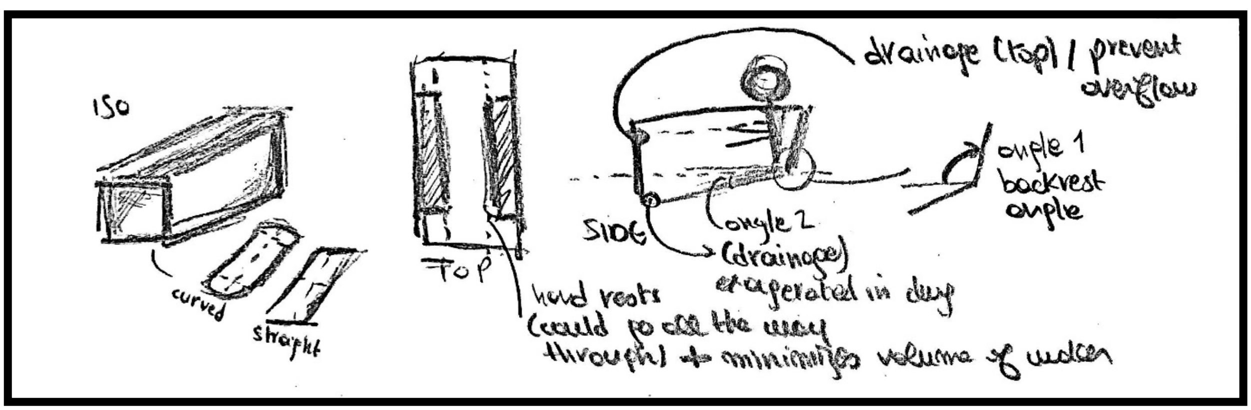


Figure 1: Initial hand sketches

2. Study

2.1. Objective

As stated in the introduction, this paper will focus on maximizing the bathtub accommodation. One of the downsides of owning a bathtub is that it requires a lot of water consumption [4]. Average usage goes from 60 to 110 gallons (227 to 416 liters), with 80 gallons (302 liters) for the "standard" bathtub [5]. Another objective needs to be incorporated to address this issue. Minimizing water usage can be achieved with a more compact design. Since wall thickness only serves for aesthetics, material consumption, and stress/load, only the inner part of the tub will get redesigned.

2.2. Target Population

It is safe to assume that the majority of people stop growing before the age of 20. Some can still grow up till their early 20s, but it is not common [6]. Since the U.S is known for its abundance of bathrooms [1], this study will target U.S citizens aged 20 and above. The NHANES 2013-2016 data [7] is the perfect population for this study. Furthermore, equal combined weights (50:50) are also applied.

Additionally, regression with residual variance is used as a data synthesis method. The linear models for each anthropometric measurement are based on the ANSUR II population [8].

2.3. Benchmarking

Various types of bathtubs are available for consumers (Alcove, corner, walk-in ...). Each one of these comes in different sizes and shapes [9]. This study does not focus on one variety, which is why benchmarking will include Alcove and Drop-in tubs since they are the most common.

(a) Alcove

(b) Drop-in

Figure 2: Types of bathtubs

Masterclass represents alcove tubs in three different sizes, with the most common size being 60 inches long, 32 inches wide, and 18 inches deep. On the other hand, drop-ins generally have a length of 60 inches, a width of 30 inches, and a depth of 16 [10].
Badeloft USA describes typical alcove tubs as 60 inches long, 30 inches wide, with 16 inches of water depth. Additionally, drop-ins would be the same size as Alcove [11].
The Spruce characterizes the standard Alcove as 60 inches in length, 32 inches in width, and 18 inches in depth. Drop-ins would be 58.5 inches long (averaged 45 to 72), 31 inches wide (averaged 30 to 32), and 17 inches deep (averaged 14 to 20) [12].
Note that these dimensions could be outside dimensions (resulting numbers might be smaller since this considers extra, unused material). All articles had more than one size (small, medium, and large), which should be combined into a single size in this study. The range is close for all three articles and two types of tubs. These numbers will compare to the numbers generated from this study later in this paper.
The following section will showcase the design variables and represent the CAD model.

2.4. Design Overview

Since this study is multivariate, accommodation means satisfying all of the variables. The list below represents the design variables:

- Basin Length
- Basin Lower Width
- Basin Height
- Armrest width
- Armrest height

* Basin Upper Width can be estimated using Basin Lower Width and Armrest Width (x2 to account for one on each side). This measurement should be similar to the “Forearm-Forearm Breadth” measurement found in ANSUR II. Another drainage hole (located on top to stop the water from overflowing) will have its location optimized in section 2.6.

The CAD model for this redesigned bathtub was done in Rhinoceros 6.0 (+Grasshopper). Design generation is prone to iterations; parametric design can help with this part. It is unlikely for a structure to be perfect forever. Newer versions/iterations are extremely common [13]. With this program (Rhino + Grasshopper), changing dimensions is made easy. A Grasshopper script has been generated (Figures 3 and 4) with number sliders that change the design variables accordingly (Figure 5).

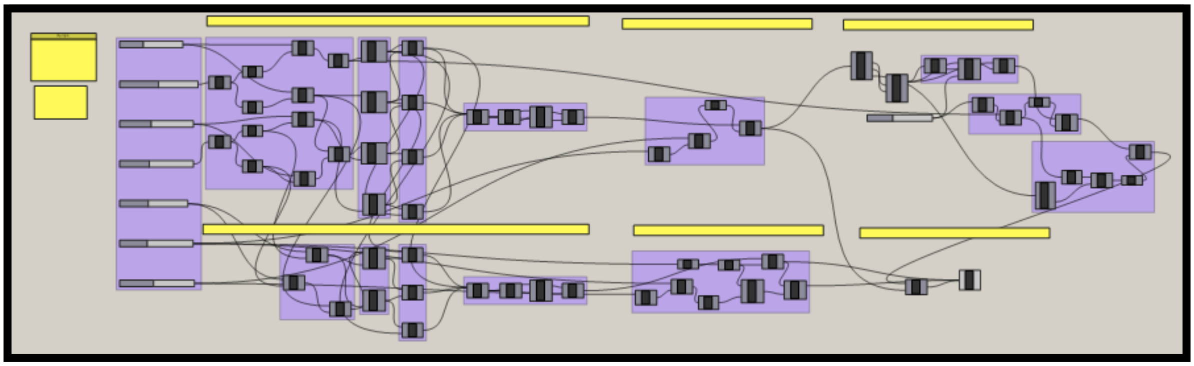


Figure 3: Grasshopper Side

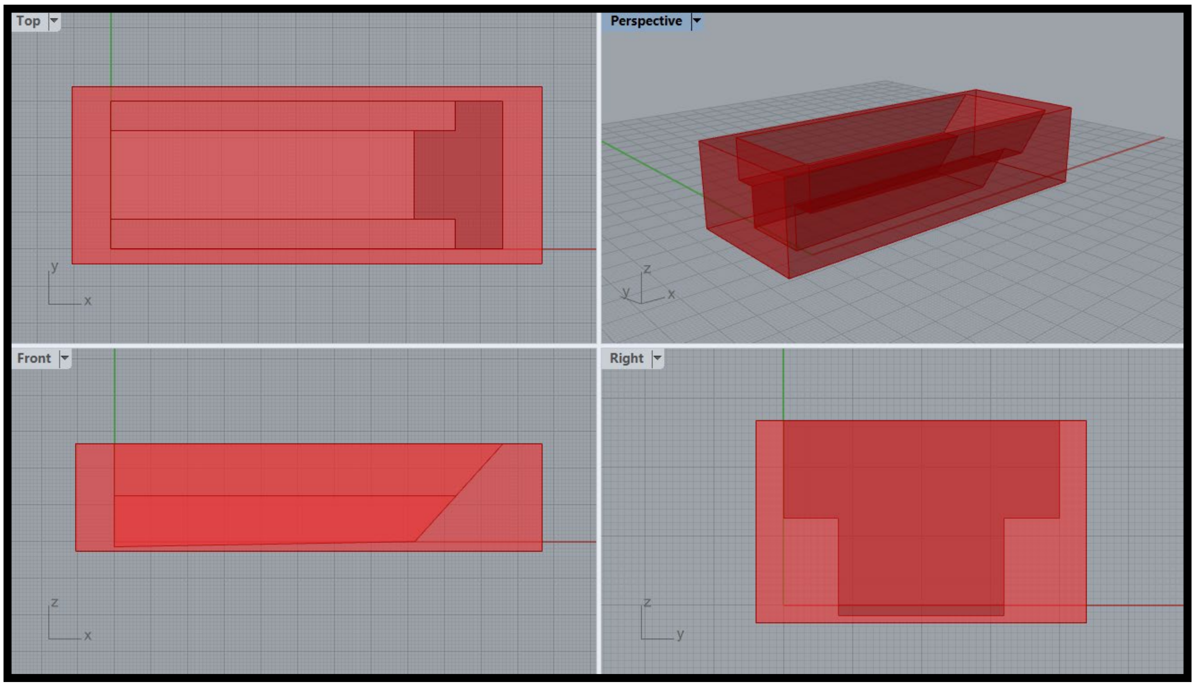


Figure 4: Rhino Side

For this model, the tub has two inclinations. The body should angle 90 degrees or more to help with comfort (on a seated position) [14], which makes 115 degrees a fair estimation for this design. Additionally, a 1.5 degrees drainage angle will help drain water smoothly when needed. This bathtub also includes armrests that extend along with the tub (which helps minimize water volume). Upcoming studies (involving different populations) could also be incorporated into this model. The rendered image for this prototype is shown in Figure 6.

In the next section of this paper, new dimensions will help maximize accommodation.

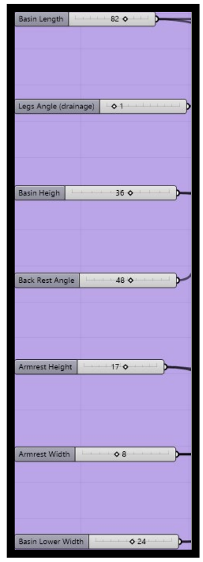


Figure 5: Grasshopper design variables sliders

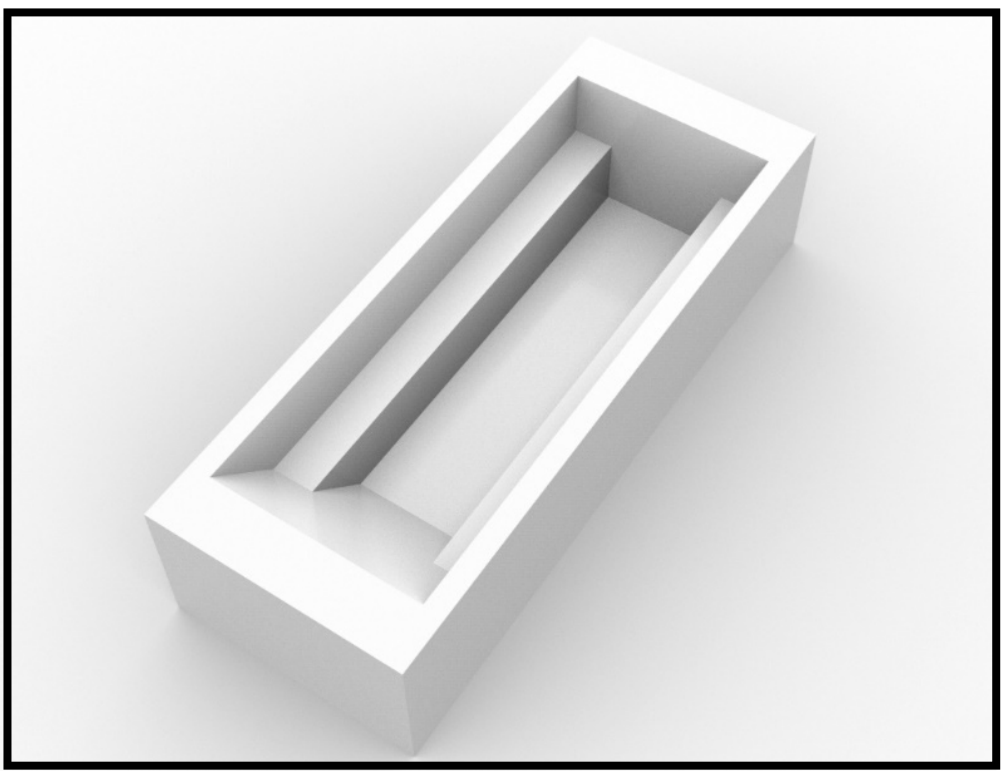


Figure 6: Bathtub Redesign Rendering

2.5. Analysis

R studio is the program of choice for this study; R is a programming language built for statistics (with various packages that aim towards simplification) [15]. As discussed in the previous section, a multivariate analysis applies to the bathtub, where accommodation is only accepted if all design variables are satisfied. Additionally, the population is weighted as 50:50 female: male. After initializing the datasets, the design variables are specified as follows:

- Functional leg length for basin length
- Seated hip breadth for basin lower width
- Seated cervical height for basin height
- Forearm circumference flexed multiplied by a constant for armrest width
- Elbow rest height for armrest height

The armrest width measurement is not available in ANSUR II. A little experiment was conducted on ten individuals to acquire it. Both forearm circumference flexed (1) and width (2) had to be measured. By dividing (2) by (1), a constant is obtained, which can then be multiplied by (1) in ANSUR II to get the final measurement (Table 1).

Table 1: Constant value for forearm width

Circumference	Width	Constant	Average
29.7	9.25	0.311448	0.324298
29.8	9.4	0.315436	
30.8	10.3	0.334416	
29.5	9.4	0.318644	
30	9.7	0.323333	
23	7.8	0.33913	
24.4	8	0.327869	
30.1	9	0.299003	
24.8	8.6	0.346774	
26	8.5	0.326923	

A correlation test is applied to each variable to build accurate linear models. Stature is the predictor for basin length and height (0.82 and 0.72 correlations); BMI helps with basin lower width and armrest width (0.81 and 0.75 correlations); Basin height works best as a predictor for the armrest height (0.62 correlation). Figure 7 represents those correlations.

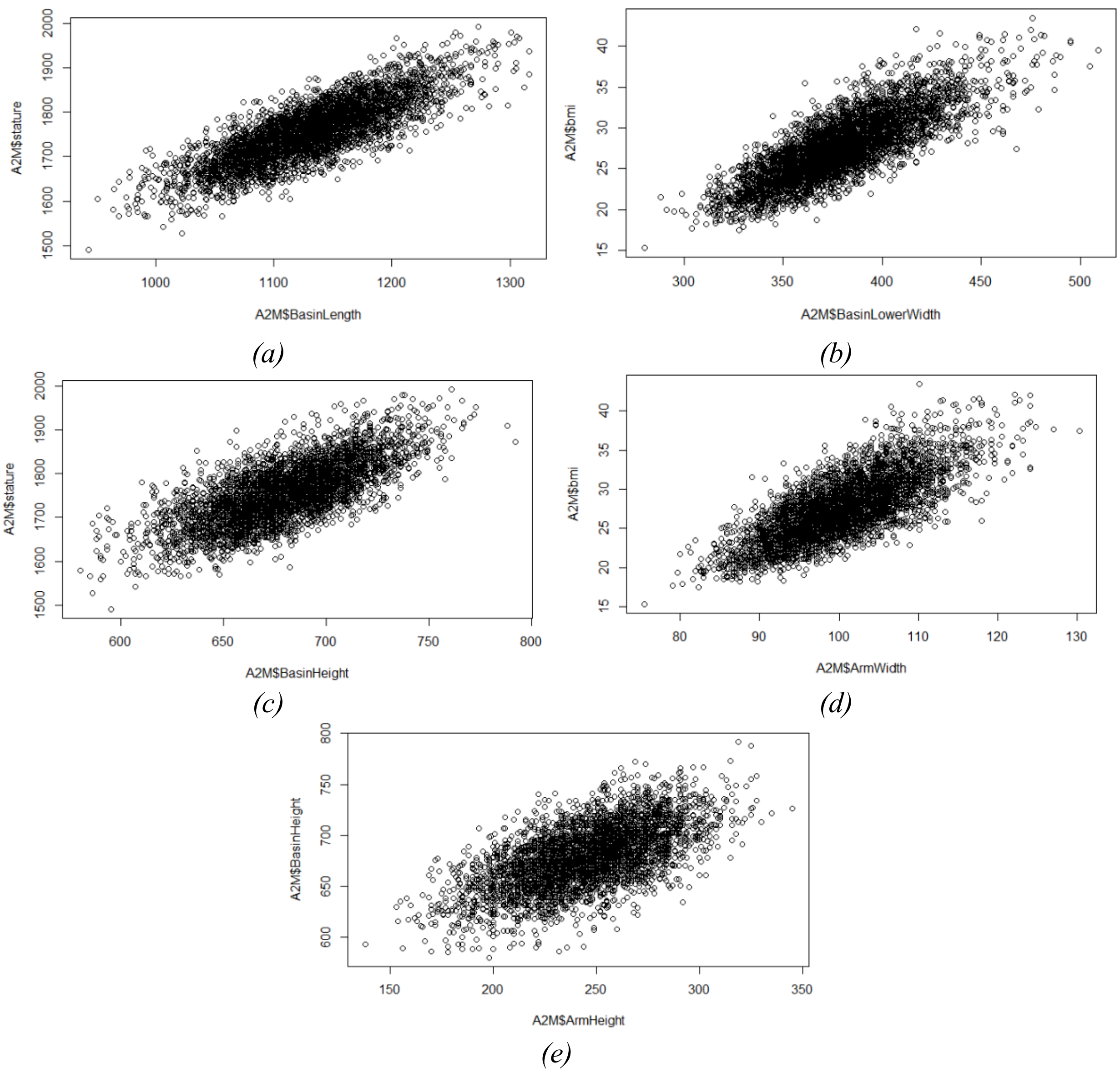


Figure 7: Correlation plots

Regression with residual variance is the data synthesis method used, following the formula below:

$$y = ax + b + N(0, s)$$

a: predictor estimate (coefficient 2 in the model)

b: estimate intercept (coefficient 1 in the model)

x: predictor

N: normal distribution (with zero mean and a standard deviation)

The reason behind going with this method (instead of proportionality constant or regular regression) is that it does the best job of modeling the overall relationship in a population.

After successfully applying this method, all the measurements will transfer to NHANES 2013-2016. Weighted quantiles are then applied to the tails, using the male or female population accordingly:

- Basin Length: Male 96th percentile (1232 mm)
- Basin Height: Male 95th percentile (731 mm)
- Basin Lower Width: Female 96th percentile (547 mm)
- Armrest width: Male 96th percentile (121 mm)
- Armrest height: Female 3rd percentile (179 mm)

All the quantiles above give a clear description of the values they represent. For basin height, the 95th percentile of males is chosen instead of the 5th percentile of females since it might cause issues with water level later. Accommodation will be extremely low, and most people will have their back sticking out of the tub. Although this problem does not sound good, other comfortability issues could occur when the head location is lower than the basin height. This topic will be discussed in the "Future Work" section.
Finally, both men and women merge into a single population with equal combined weights (to account for 50:50). Calculations result in a 90% accommodation (extreme tails disaccommodated only). Table 2 shows all necessary values.
The upcoming section discusses an optimized location for the top drainage hole.

Table 2: Accommodations

		Combined Population (50:50)					
Males	Females	Basin Length	Basin Lower Width	Basin Height	Armrest Width	Armrest Height	Overall
87.27	92.74	97.92	97.87	97.4	97.18	97.8	89.99

2.6. Optimization

Optimization problems consist of finding the most efficient way of using confined resources to reach a specific objective (maximizing/minimizing). A mathematical model is built using the following components [16]:

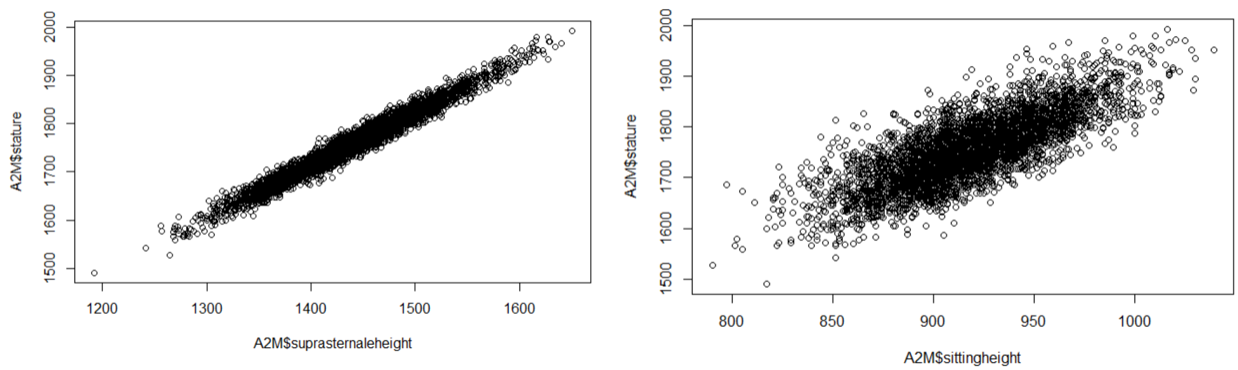
- Design variables: values that can be altered.
- Objective function: what needs optimization.
- Constraints: similar to boundaries or restrictions.

Gradient-based optimization algorithms will generally find the closest local minimum to the starting point. It is inadequate for exploring the design space and looking for other regions.

Alternatively, stochastic global optimization algorithms might give different answers each run since different pathways are followed. They follow heuristic/metaheuristic methods and guarantee a global minimum [17]. In this paper, the Microsoft Excel solver acts as the optimizer (evolutionary used as a solving method to represent global instead of gradient). Multi start (a technique where the starting values for the design variables are changed) is also employed. An additional anthropometric measure should act as the maximum water level for every individual. Suprasternal sitting height (ssh) applies to this case.

$$ssh = suprasternal\ height - (stature - sitting\ height)$$

Since ssh had low correlations, both suprasternal height and sitting height got synthesized (regression + residual variance) into NHANES where the above calculations happened. 0.98 and 0.78 correlations are represented in Figures 8a and 8b respectively.



(a) *(b)*

Figure 8: Correlation plots

Since minimizing water usage acts as our optimization's primary objective, water volume follows the formula below:

$$Total\ Water\ Volume = Tub(no\ handles) - Tub\ handles - Individual$$

With the quantile values from section 2.5, the Tub (no handles) and Tub handles volumetric measurements are calculated. All numbers are constant except for Basin Height which is replaced with the Water Level (ssh) variable (Appendix A shows all the spreadsheets for those calculations). Table 3 helps with the volume of each individual, but only the mean of those values is used (this value acts as a constant value).

Table 3: Human Volume

Parameter	Equation	
% body fat (men)	$\%bf = (1.2 \times BMI) + (0.23 \times age) - 16.2$	[18]
% body fat (women)	$\%bf = (1.2 \times BMI) + (0.23 \times age) - 5.4$	[18]
Body density	$d\left({}^{kg}/_{m^3}\right) = \dfrac{495}{\%bf + 450}$	[19]

Volume	$V(m^3) = {}^{m(kg)}/_{d}$

The only design variable, in this case, is the top drainage hole location (also called water level or ssh); this value is different for each individual. This optimization is univariate since the bathtub dimensions have already been settled previously; what is left is the top drainage location. The objective function is minimizing water volume. This study has to be under specific constraints:

- Minimum water level: which is also ssh 5th percentile (504 mm)
- Maximum water level: basin height (731 mm). These values help the solver, and they also fit this analysis.
- Minimum accommodation: 90% (people fit this model if their ssh < design variable)
- Maximum volume: for this value, benchmarking is important (Averaging the values from section 2.3 (for both types of tubs)). Note that this applies to medium tubs. Since this redesigned tub might end up somewhere between the medium and large versions, the following equation applies (where 1/3 is an arbitrary value):

$$\begin{aligned} Total\ Water\ Volume\ (Benchmark) \\ &= Avg.med.benchmark\ volume \\ &+ {}^{1}/_{3}\,(Avg.large\ benchmark\ volume \\ &-\ Avg.med.benchmark\ volume) - Individual \end{aligned}$$

Since it is unclear if benchmark dimensions are inner or outer, we will assume they are internal for the scope of this study. This value (Max. Volume) ends up as 0.502 m^3.

Finally, after running the optimization using Microsoft Excel solver, the results are shown in Table 4. The redesigned tub has a volume slightly larger than average medium sizes but smaller than large ones (also accommodating more than 90% of the population).

Table 4: Optimization Results

Design Variables	Water Level	629
Objective	total volume (m^3/shower)	0.493166
Constraints	Level min	504
	Level max	731
	Max volume	0.502309
	Min accommodation	90.00%

accommodation	90.25%

2.7. Long Life Product

The configuration for secondary markets and future populations (also called long-life products) is a crucial aspect of design. Summary statistics are not enough; data synthesis methods (regression + residual variance) produce better results. When working with long-life products, two things are considered:

- Secular trends: shifts in body size and shape in a population over time
- Changing demographics: different races, genders...

Anticipation is essential; how can the chosen population change? The two cases below take this into account:

Table 5: Accommodations for 70:30 female:male

		Combined Population (70:30)					
Males	Females	Basin Length	Basin Lower Width	Basin Height	Armrest Width	Armrest Height	Overall
87.27	92.74	98.67	97.04	98.49	97.69	97.38	91.02

Table 6: Accommodations for 30:70 female:male

		Combined Population (30:70)					
Males	Females	Basin Length	Basin Lower Width	Basin Height	Armrest Width	Armrest Height	Overall
87.27	92.74	97.1	98.62	96.17	96.34	98.01	88.41

With the chosen measurements in section 2.5, the percentage levels are seen in tables 5 and 6. Table 5 shows a higher overall accommodation than Table 2 (the only values that went down were basin lower width and armrest height because their quantiles depend on females). The exact opposite happened in Table 6.

The overall accommodations are still close to 90% in both cases. This means that the overall dimensions of the bathtub are suitable. The results are also straightforward and expected.

3. Conclusion

3.1. Reflection

The goal of this study is to redesign the standard bathtub. When redesigning, the new product should be better than the old one. The comparison between the benchmarks and the redesigned bathtub is located in the table below:

Table 7: Results and Comparison

	Avg. Medium	Avg. Large	Redesign
Basin Length *(in)*	59.75	72	48.5
Basin Width *(in)*	30.83	34.8	31.06
Basin Height *(in)*	16.83	19.6	28.78
Overall Volume (Empty Tub) (m^3)	0.508	0.805	0.574
Max Water Volume (m^3)	0.427	0.723	0.493
Accommodation Length *(%)*	100	100	97.92
Accommodation Width *(%)*	99.99	100	97.87
Accommodation Height *(%)*	0	0	97.4
Total Accommodation *(%)*	0	0	89.99

For Height to be accommodated, the individual's sitting cervical height should be smaller than the Basin Height (for this redesign). The way the benchmarks are using this measurement is the opposite (bigger than Basin Height accommodates the individual). In this case, it would result in a 100% accommodation (check section 3.2 for more details). The reason this criteria was changed for this paper is for maximum water level purposes. In the optimization section, this measurement (water level or ssh) has been determined so that 90.25% of the population can get water up to their suprasternale sitting height. This would make more sense as every individual with a ssh below the maximum would be accommodated. Not a single person would be able to do that in the benchmarks (even in the large tub).
For the "Redesign", Basin Width is the overall width (including armrests). Additionally, the redesigned accommodation involves two other design variables (armrest height and width) . Accommodation is for NHANES 2013-2016 50:50 weighted men: women population. The Max Water Volume includes the individual sitting in the tub.
Although univariate accommodations are very similar (assuming non-zero for medium and large), the maximum water volume for the new tub is close to the medium and far from the large. Furthermore, it has been deigned to fit 89.99% of the population "comfortably".

3.2. Future Work

For the scope of this study, the benchmarks were assumed as inner dimensions. With the accommodations retrieved, some numbers seemed unsuitable. For this specific reason, the next iteration will incorporate more accurate measurements (for benchmark). As discussed earlier, the bathtub height is drastically bigger than all benchmarks; for the first prototype, a headrest (made of waterproof, pillow-like material) would go in the middle of the backrest all the way to the basin height (731mm). Additionally, it would accommodate 100% of people if it goes out of the tub to a specified distance. A new CAD model would also have manufacturability as a new requirement.

This study used the manikin approach (does not include preference). The next iteration would integrate changes using the hybrid approach (manikin + population). For this to be done, live measurements would be included instead of anthropometric data where needed (to take preference into account, and make it reconfigurable).

4. References

[1] Thompson, Derek. "America Is Overrun with Bathrooms." The Atlantic. Atlantic Media Company, January 23, 2020. https://www.theatlantic.com/ideas/archive/2020/01/why-do-american-houses-have-so-many-bathrooms/605338/.

[2] Lee, Soomin, Shogo Ishibashi, Yoshihiro Shimomura, and Tetsuo Katsuura. "Physiological Functions of the Effects of the Different Bathing Method on Recovery from Local Muscle Fatigue." Journal of Physiological Anthropology 31, no. 1 (2012). https://doi.org/10.1186/1880-6805-31-26.

[3] Goto, Yasuaki, Shinya Hayasaka, Shigeo Kurihara, and Yosikazu Nakamura. "Physical and Mental Effects of Bathing: A Randomized Intervention Study." Evidence-Based Complementary and Alternative Medicine 2018 (2018): 1–5. https://doi.org/10.1155/2018/9521086.

[4] "The Pros and Cons of Showers vs Tubs." Scott McGillivray, March 23, 2021. https://scottmcgillivray.com/showers-and-tubs-pros-and-cons/.

[5] Badeloft. "How Many Gallons Does a Bathtub Hold?" Badeloft, October 29, 2019. https://www.badeloftusa.com/ideas/how-much-does-bathtub-hold/.

[6] "When Do You Stop Growing?" Colorado Primary Health Care, February 17, 2016. https://coloradoprimaryhealthcare.com/2014/06/when-do-you-stop-growing/.

[7] "Nhanes." The OPEN Design Lab. Accessed November 14, 2021. https://www.openlab.psu.edu/nhanes/.

[8] "Ansur II." The OPEN Design Lab. Accessed November 14, 2021. https://www.openlab.psu.edu/ansur2/.

[9] "Types of Bathtubs." The Home Depot. Accessed November 21, 2021. https://www.homedepot.com/c/ab/types-of-bathtubs/9ba683603be9fa5395fab90209ab53e.

[10] MasterClass. "Guide to Bathtub Sizes: 8 Common Types of Bathtubs - 2021." MasterClass. MasterClass, June 4, 2021. https://www.masterclass.com/articles/bathtub-sizes-explained#8-types-of-bathtubs.

[11] Badeloft. "Standard Bathtub Dimensions for Every Type of Tub." Badeloft, June 17, 2021. https://www.badeloftusa.com/ideas/bathtub-dimensions/.

[12] Wallender, Lee. "Standard Bathtub Sizes: Reference Guide to Common Tubs." The Spruce. The Spruce, May 5, 2021. https://www.thespruce.com/bathtub-sizes-reference-guide-1821342.

[13] Barrios Hernandez, Carlos Roberto. "Thinking Parametric Design: Introducing Parametric Gaudi." Design Studies 27, no. 3 (2006): 309–24. https://doi.org/10.1016/j.destud.2005.11.006.

[14] Hawks, Leona. "Selecting a Comfortable Chair." Utah State University Extensions, 1987.

[15] Paruchuri, Vik. "Six Reasons Why You Should Learn R for Data Science." Dataquest, June 16, 2021. https://www.dataquest.io/blog/three-mighty-good-reasons-to-learn-r-for-data-science/.

[16] Chandrakantha, Leslie. "Using Excel Solver in Optimization Problems," n.d., 42–49.

[17] Gomez, Jonatan. "Stochastic Global Optimization Algorithms: A Systematic Formal Approach." Information Sciences 472 (2019): 53–76. https://doi.org/10.1016/j.ins.2018.09.021.

[18] LaMeaux, E.C. "How to Calculate Your Ideal Body Fat Percentage." Gaiam. Accessed December 8, 2021. https://www.gaiam.com/blogs/discover/how-to-calculate-your-ideal-body-fat-percentage.

[19] Wood, Robert. "Siri Equation." Topend Sports, science, training and nutrition. Accessed December 8, 2021. https://www.topendsports.com/testing/siri-equation.htm.

5. Appendix

A. Volume Spreadsheets

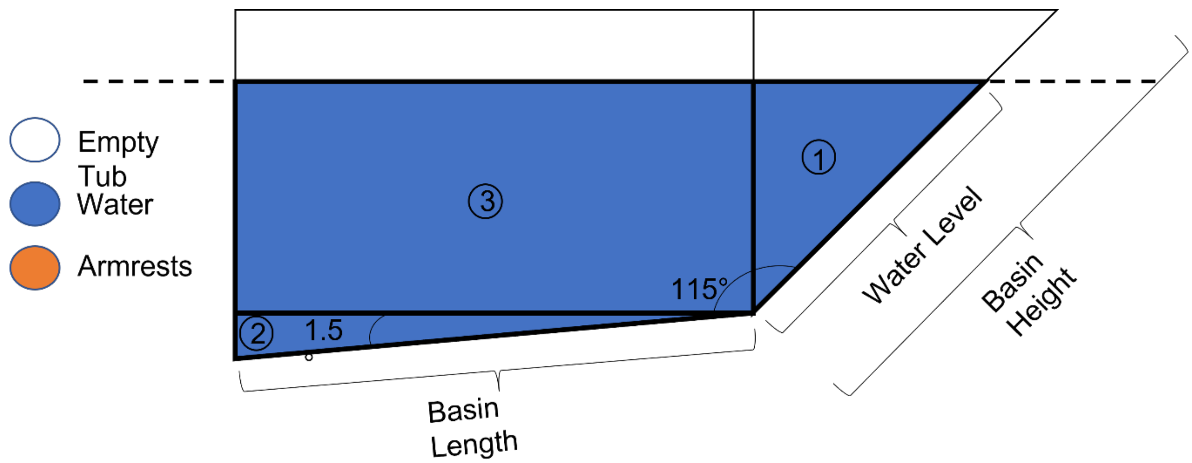


Figure 9: Empty tub, no armrests

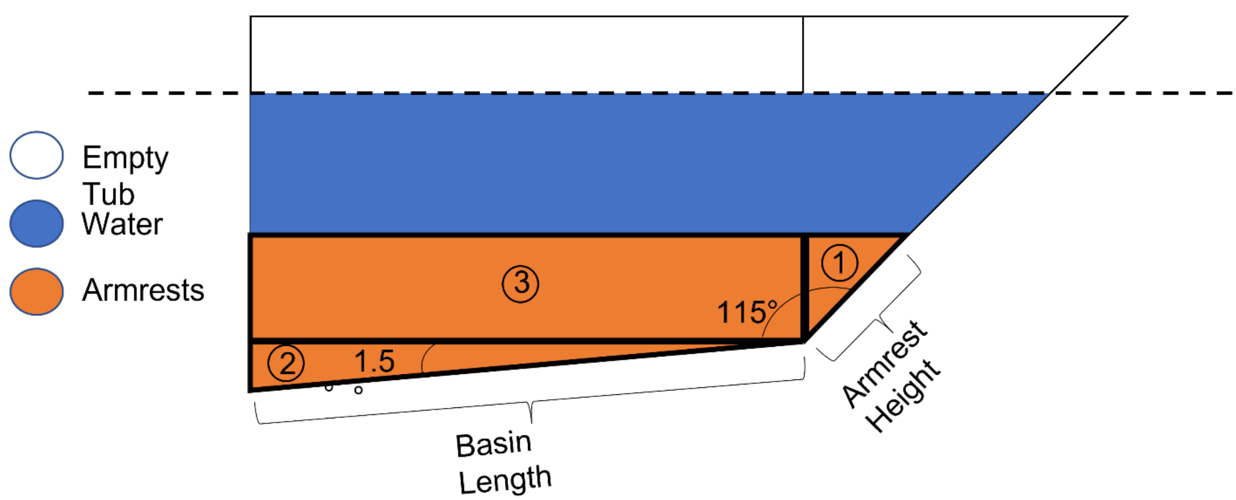


Figure 10: Empty tub (Only armrests)

Table 8: Volume calculations for "Empty tub, no armrests"

Right Triangle (1)	Empty tub, no armrests		
Angle (rad)	0.436332313	25	deg
Water Level (hyp)	629	mm	
Top	265.8268866	mm	
Left	570.067598	mm	
Area	0.075769647	m^2	
Right Triangle (2)			
Angle (rad)	0.026179939	1.5	deg
Basin Length (hyp)	1232	mm	
Top	1231.577824	mm	
Left	32.25000032	mm	
Area	0.019859193	m^2	
Middle Rectangle (3)			
First	570.067598	mm	
Second	1231.577824	mm	
Area	0.702082612	m^2	
Total Area	0.797711452	m^2	
Basin Lower Width	0.547	547	
Armrest Width	0.121	121	
Depth	0.789	m	
Overall Volume (Empty tub, no armrests)	0.629394336	m^3	

Table 9: Volume calculations for "Only armrests"

Right Triangle (1)	Only armrests		
Angle (rad)	0.436332313	25	deg
Armrest Height	180	mm	
Top	76.07128711	mm	
Left	163.1354017	mm	
Area	0.00620496	m^2	
Right Triangle (2)			
Angle (rad)	0.026179939	1.5	deg
Basin Length	1232	mm	
Top	1231.577824	mm	
Left	32.25000032	mm	
Area	0.019859193	m^2	
Middle Rectangle (3)			
First	163.1354017	mm	
Second	1231.577824	mm	
Area	0.200913943	m^2	

Total Area	0.226978096	m^2
Armrest Width	0.121	121
Depth	0.242	m
Overall Volume (Only armrests)	0.054928699	m^3

Table 10: Volume calculations for "Final Redesigned Volume"

Empty tub (With armrests)	0.574465637	m^3
Human volume	0.0813	m^3
Final volume	0.493165637	m^3

Table 11: Dimensions chosen for benchmarks for comparison

Benchmarking Dimensions		
avg (med)	avg (large)	one third
59.75	72	63.8
30.83	32	31.22
16.83	20	17.88

Table 12: Volume calculations for "Old Benchmark Volume"

old (from benchmarking)				
Basin Length	1620.52	mm	63.8	in
Basin Width	792.988	mm	31.22	in
Basin Height	454.152	mm	17.88	in
Overall Volume (Empty tub)	0.583609351	m^3		
Final Volune	0.502309351	m^3		